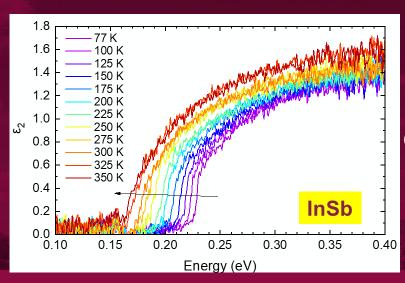


Excitonic absorption in semiconductors with low and high carrier densities



Stefan Zollner

With:

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Jose Menendez (Arizona State University, Tempe, AZ)

Email: zollner@nmsu.edu. WWW: http://femto.nmsu.edu.

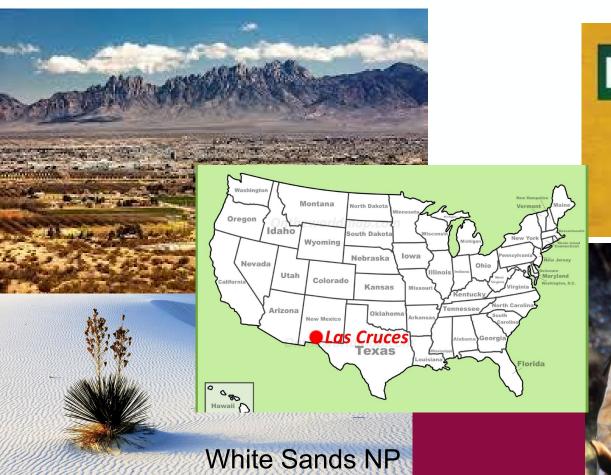


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College of Arts and Sciences, Department of Physics New Mexico State University, Las Cruces, NM, USA



Where is Las Cruces, NM???







New Mexico State University, Las Cruces



Land grant institution, Carnegie R2 (soon to be R1)

Comprehensive: Arts and Sciences, Education, Business, Agriculture Ph.D. programs in sciences, engineering, agriculture; Ag extension

14,000 students (11,500 UG, 2,500 GR), 1000 faculty

Minority-serving, Hispanic-serving (60% Hispanic/NA, 26% White) Small-town setting



Military-friendly institution (Army and Air Force ROTC programs)

Community engagement classification (first-generation students, Pell grant recipients)

Physics: BS/BA, MS, PhD degrees. 71 UG and 37 GR students.12 faculty (HE Nuclear and Materials Physics), 1.7 M\$ expenditures.ABET-accredited BS in Physics and BS in Engineering Physics

Ellipsometry at NMSU



M Bauer, J Taraci, J Tolle, AVG Chizmeshya, S Zollner, DJ Smith, ...

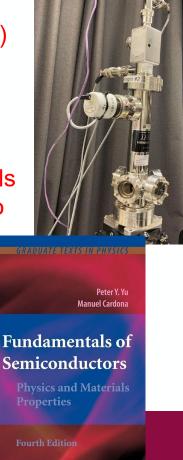
Applied physics letters 81 (16), 2992-2994

Ellipsometry on anything (inorganic, 3D)

- Metals, insulators, semiconductors
- Mid-IR to vacuum UV
- 10 to 800 K

Ellipsometry tells us a lot about materials quality (not necessarily what we want to know).

Optical critical points of thin-film Ge_{1−y}Sn_y alloys: A comparative Ge_{1−y}Sn_y / Ge_{1−x}Si_x 429 2006 study
 VR D'costa, CS Cook, AG Birdwell, CL Littler, M Canonico, S Zollner, ...
 Physical Review B 73 (12), 125207
 Growth and strain compensation effects in the ternary Si_{1−x−y}Ge_xC_y alloy system 404 1992
 K Eberl, SS lyer, S Zollner, JC Tsang, FK LeGoues
 Applied physics letters 60 (24), 3033-3035
 Ge—Sn semiconductors for band-gap and lattice engineering 322 2002



♠ Springer

Problem statement

- (1) Achieve a quantitative understanding of absorption and emission processes.
- Our <u>qualitative</u> understanding of excitonic absorption is 50-100 years old,
- But **insufficient** for modeling of detectors and emitters.
- (2) How are optical processes affected by high carrier concentrations (screening)?
- High carrier densities can be achieved with
 - In situ doping or
 - ultrafast (femtosecond) lasers or
 - high temperatures (narrow-gap or gapless semiconductors)
- Goal: CMOS-integrated mid-infrared camera (thermal imaging with a phone).
- Future: How are optical processes affected by an electric field (pin diode or thin layer)?



Intensity of Optical Absorption by Excitons

R. J. Elliott

Phys. Rev. 108, 1384 - Published 15 December 1957

Article References

Citing Articles (1,780)

PDF

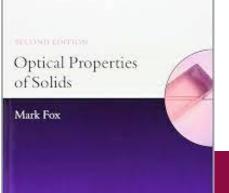
Export Citation



ABSTRACT

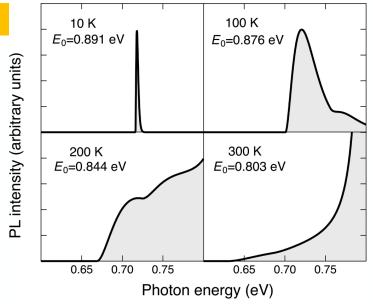
The intensity of optical absorption close to the edge in semiconductors is examined using band theory together with the effective-mass approximation for the excitons. Direct transitions which occur when the band extrema on either side of the forbidden gap are at the same \mathbf{K} , give a line spectrum and a continuous absorption of characteristically different form and intensity, according as transitions between band states at the extrema are allowed or forbidden. If the extrema are at different \mathbf{K} values, indirect transitions involving phonons occur, giving absorption proportional to $(\Delta E)^{\frac{1}{2}}$ for each exciton band, and to $(\Delta E)^2$ for the continuum. The experimental results on $\mathrm{Cu}_2\mathrm{O}$ and Ge are in good qualitative agreement with direct forbidden and indirect transitions, respectively.

Received 9 April 1957

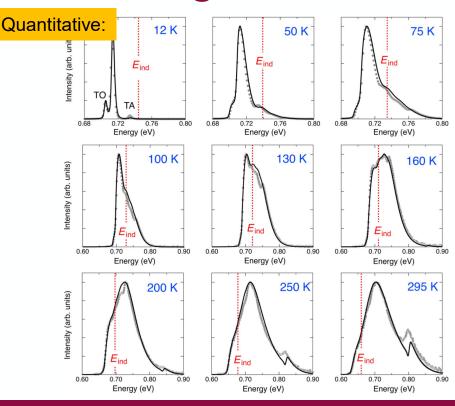


Example 1: photoluminescence in germanium

Qualitative:

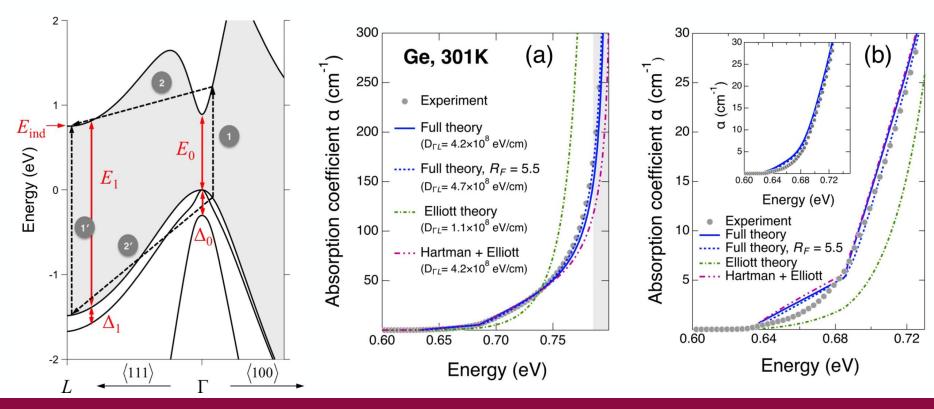


Roosbroeck-Shockley equation (PR **94**, 1558, 1954). Complete failure above **200** K and at RT.





Example 2: indirect absorption in germanium



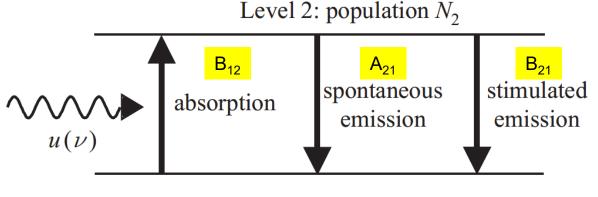


Optical constants of highly excited semiconductors

- Einstein coefficients, Fermi's Golden Rule, Elliott-Tanguy excitons
- Direct gap absorption in germanium from 10 to 800 K
- Optical constants of <u>highly excited semiconductors</u>
 - Direct gap absorption in InSb from 10 to 800 K
 - Intravalence band absorption in topological insulators (α -tin)
 - Optical constants of highly excited germanium (femtosecond ellipsometry at ELI Beamlines in Prague)
- Conclusion and Outlook



Einstein coefficients



Level 1: population N_1

In equilibrium: N₁, N₂ constant.

Absorption and emission balance.

Black-body radiation $u(\hbar\omega)$

$$B_{12}N_1u(\hbar\omega) = A_{21}N_2 + B_{21}N_2u(\hbar\omega)$$

One coefficient is sufficient:

$$g_1 B_{12} = g_2 B_{21}$$

$$A_{21} = \frac{2\hbar\omega^3}{\pi c^3} B_{21}$$

Use Fermi's Golden Rule to calculate B₁₂



Fermi's Golden Rule: Tauc plot

Direct band gap absorption

$$\frac{1}{\tau} = \frac{2\pi}{\hbar} \int_{i,f} |\langle f | H_{eR} | i \rangle|^2 \delta \left(E_f - E_i - \hbar \omega \right) = \frac{2\pi}{\hbar} |\langle f | H_{eR} | i \rangle|^2 g_{fi}(\hbar \omega) \quad \text{Joint DOS parabolic bands}$$

$$\langle f|H_{eR}|i\rangle = \frac{e}{m_0} \langle f|\overrightarrow{p}|i\rangle \cdot \overrightarrow{A}_0$$

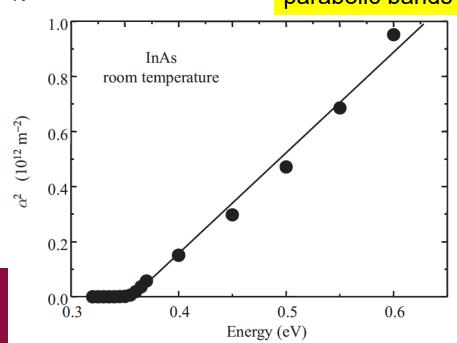
Use $\mathbf{k} \cdot \mathbf{p}$ matrix element P: $E_P = 2P^2/m_0$

Use
$$\mathbf{k} \cdot \mathbf{p}$$
 matrix element P : $E_p = 2P^2/m_0$

$$\varepsilon_2(\hbar\omega) = \frac{e^2\sqrt{m_0}\mu^{\frac{3}{2}}}{3\pi\sqrt{2}\varepsilon_0\hbar} \frac{E_P\sqrt{E_0}}{(\hbar\omega)^2} \sqrt{\frac{\hbar\omega}{E_0}} - 1$$

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constant k·p matrix element



Fermi's Golden Rule: Tauc plot

Direct band gap absorption

 $\frac{1}{\tau} = \frac{2\pi}{\hbar} \int_{i,f} |\langle f | H_{eR} | i \rangle|^2 \delta(E_f - E_i - \hbar \omega) = \frac{2\pi}{\hbar} |\langle f | H_{eR} | i \rangle|^2 g_{fi}(\hbar \omega)$

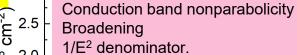
$$\langle f|H_{eR}|i\rangle = \frac{e}{m_0} \langle f|\overrightarrow{p}|i\rangle \cdot \vec{A}_0$$

Use $\mathbf{k} \cdot \mathbf{p}$ matrix element P: $E_P = 2P^2/m_0$

$$\varepsilon_{2}(\hbar\omega) = \frac{e^{2}\sqrt{m_{0}}\mu^{\frac{3}{2}}}{3\pi\sqrt{2}\varepsilon_{0}\hbar} \frac{E_{P}\sqrt{E_{0}}}{(\hbar\omega)^{2}} \sqrt{\frac{\hbar\omega}{E_{0}} - 1}$$

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0.1

1.0

0.5

0.0

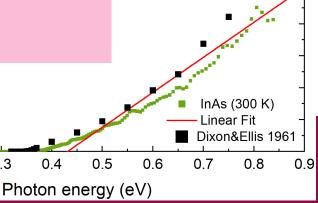
0.0

excitons

0.2

No linear region:

The Tauc plot is wishful thinking.

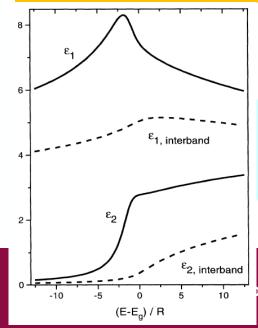


Elliott-Tanguy exciton absorption

Direct band gap absorption

Excitonic binding energy: $R=R_H \times \mu_h / \epsilon_s^2$

$$\varepsilon_{2}(\hbar\omega) = \frac{e^{2}\sqrt{m_{0}}\mu^{\frac{3}{2}}}{3\pi\sqrt{2}\varepsilon_{0}\hbar}\frac{E_{P}\sqrt{R}}{(\hbar\omega)^{2}}\left[\sum_{n=1}^{\infty}\frac{4\pi R}{n^{3}}\delta\left(\hbar\omega - E_{0} + \frac{R}{n^{2}}\right) + \frac{2\pi H(\hbar\omega - E_{0})}{1 - \exp\left(-2\pi\sqrt{R/\hbar\omega - E_{0}}\right)}\right]$$

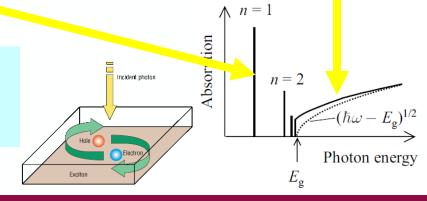


bound excitons

exciton continuum enhancement

Tanguy's contributions:

- Add Lorentzian broadening
 - Kramers-Kronig transform to get the real part.

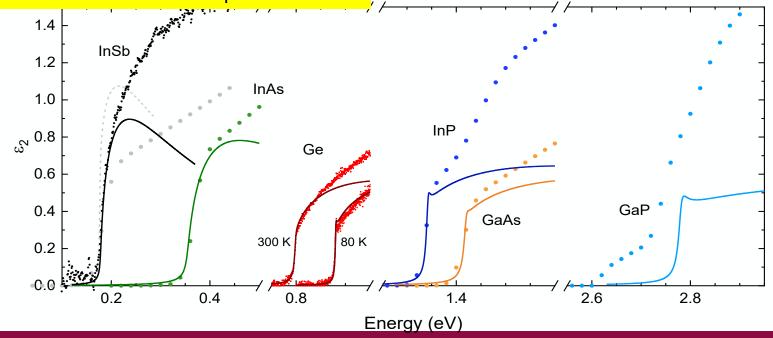


e the Future.

R. J. Elliott, Phys. Rev. **108**, 1384 (1957). Christian Tanguy, Phys. Rev. Lett. **75**, 4090 (1995) + (E)

Calculation of absorption spectrum from k·p theory

Can we calculate the absorption spectrum? Yes, we can for Ge in the low carrier density limit. It does not work for other III/V compounds.

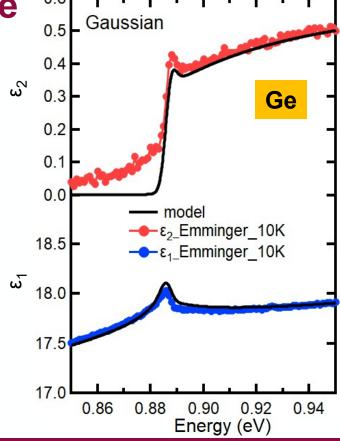




Elliott-Tanguy theory applied to Ge

- Fixed parameters:
- Electron and hole masses (temperature dependent)
- Excitonic binding energy R
- Amplitude A (derived from matrix element P)
- Adjustable parameters:
- Broadening Γ: 2.3 meV
- Band gap E₀
- Linear background A₁ and B₁ (contribution from E₁ to real part of ε)
- · Problems:
- Broadening below the gap (band tail, oxide correction)

Quantitative agreement



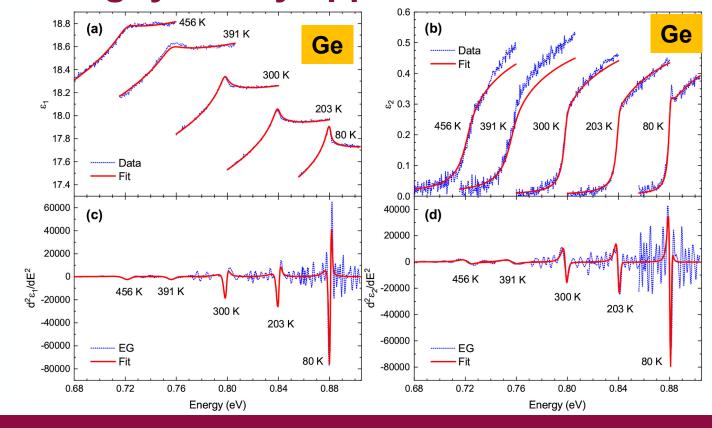
Elliott-Tanguy theory applied to Ge

Good agreement at low temperatures.

Model also describes second derivatives.

Potential problems:

- Matrix element kdependent
- Nonparabolicity
- Resonant indirect absorption
- ??? at high T.





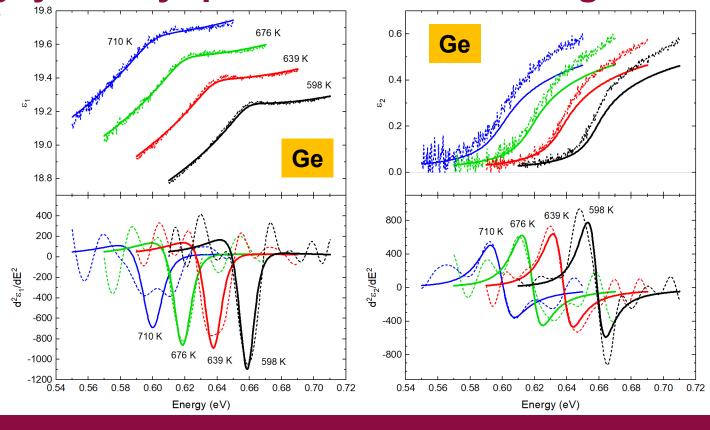
Elliott-Tanguy theory: problems for Ge at high T

Good agreement at low temperatures.

Model also describes second derivatives.

Potential problems:

- Matrix element kdependent
- Nonparabolicity
- Resonant indirect absorption
- Temperature dependence of the effective mass.





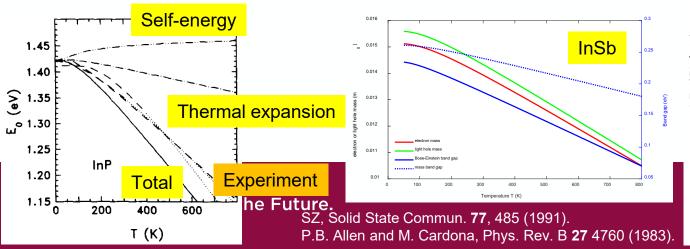
Temperature dependence of the effective mass

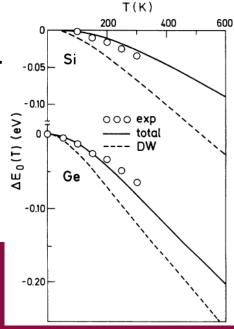
Effective electron mass given by k⋅p theory

$$\frac{1}{m_e(T)} = 1 + \frac{E_P}{3} \left(\frac{2}{E_0(T)} + \frac{1}{E_0(T) + \Delta_0} \right)$$

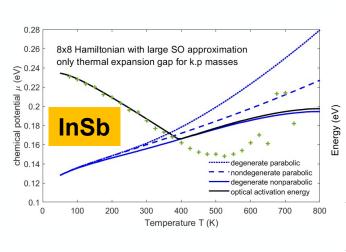
 $\mathbf{E_0}$: direct band gap $\mathbf{k} \cdot \mathbf{p}$ matrix element P: $E_P = 2P^2/m_0$

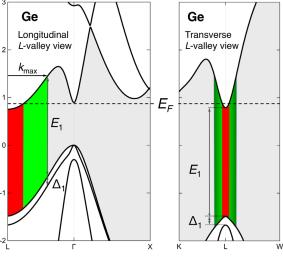
- Temperature dependence of the direct band gap has two contributions:
 - Thermal expansion of the lattice
 - Electron-phonon scattering (Debye-Waller term and self-energy)
- "Mass band gap" should only include the thermal expansion.

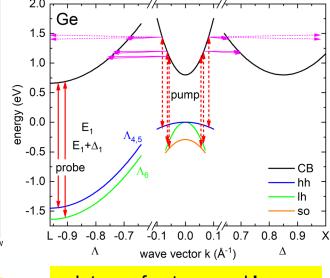




Optical Absorption at High Carrier Densities







High temperature (thermal excitation of e-h pairs) constant m and E_0

High n-doping of Ge with P (free electrons pile up at L-point)

Intense femtosecond laser excitation (ELI Beamlines) (electrons pile at L-point)

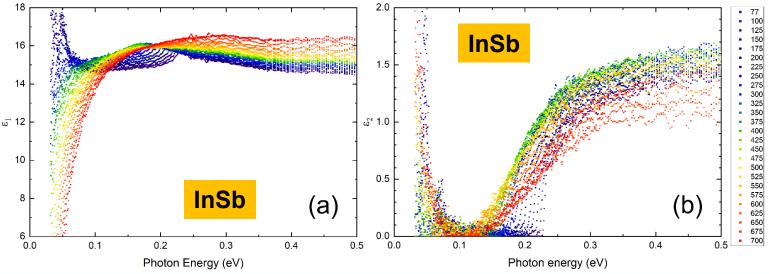
Rivero, JVSTB 41, 022203 (2023)

Xu et al., PRL **118**, 267402 (2017)

Espinoza, APL **115**, 052105 (2019)



(1) Dielectric function of InSb from 80 to 800 K





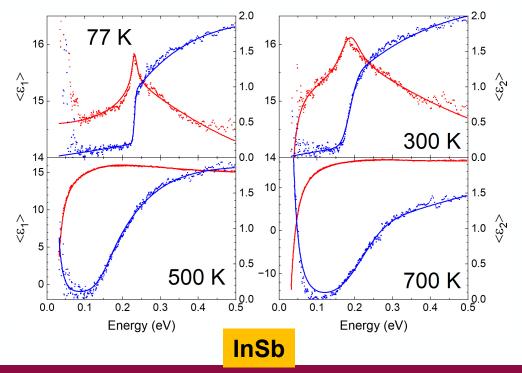
- Band gap changes with temperature (but only below 500 K).
- Amplitude reduction at high temperatures (Pauli blocking, bleaching)
- Drude response at high temperatures (thermally excited carriers).
- Depolarization artifacts at long wavelengths (below 300 K).

Woollam FTIR-VASE cryostat with CVD diamond windows

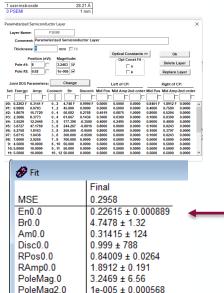


Band gap analysis for InSb

How does the band gap of InSb change with temperature?



Parametric-Semiconductor Model:



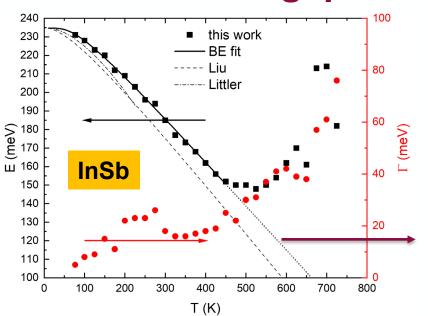
Also vary "shape parameters".

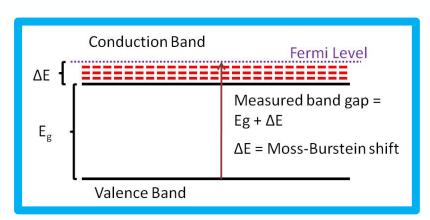
Asymmetric peak shape poorly described.

Try Tanguy oscillator for excitonic line shape.



Band gap of InSb from 80 to 800 K





Bose-Einstein Model

$$E_0(T) = E^{\text{un}} - b \left[1 + \frac{2}{\exp\left(\Omega/k_B T\right)} \right]$$

- Band gap changes with temperature (but only below 500 K)
- Described by Bose-Einstein model below 500 K: Logothetidis, PRB 31, 947 (1985).
- No redshift above 500 K: Thermal Burstein-Moss shift



k⁻p theory (band structure method)

Schrödinger equation

$$H\Phi_{n\vec{k}} = \left(\frac{\vec{p}^2}{2m_0} + V\right)\Phi_{n\vec{k}} = E_{n\vec{k}}\Phi_{n\vec{k}}$$

Use Bloch's theorem:

$$\Phi_{n\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}u_{n\vec{k}}(\vec{r})$$
 Product rule
$$(fg)'' = f''g + 2f'g' + fg''$$

Solve equation for k=0. $\left(\frac{\vec{p}^2}{2m_0} + \frac{\hbar^2 \vec{k}^2}{2m_0} + \frac{\hbar \vec{k} \cdot \vec{p}}{m_0} + V \right) u_{n\vec{k}} = E_{n\vec{k}} u_{n\vec{k}}$

Eliminate green free-electron term with substitution of variables (Kane 1957). Then treat red term in perturbation theory.

Works very well for semiconductors with local V(r) potentials.



Nonparabolicity of InSb conduction band from k-p theory

E₀=0.237 eV

 $m_e^{-0.0152}$

 $m_{lh} = 0.0156$

0.2

0.3

Energy above CB minimum or below VB maximum (eV)

0.5

0.6

0.7

square of t 6.0 0.0

Kane 8x8 k·p Hamiltonian:

$$\widetilde{H}_{\vec{k}} = \begin{pmatrix} E_0 & 0 & -\frac{\hbar \vec{k}}{m_0} iP & 0 \\ 0 & -\frac{2\Delta_0}{3} & \frac{\sqrt{2}\Delta_0}{3} & 0 \\ \frac{\hbar \vec{k}}{m_0} iP & \frac{\sqrt{2}\Delta_0}{3} & -\frac{\Delta_0}{3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

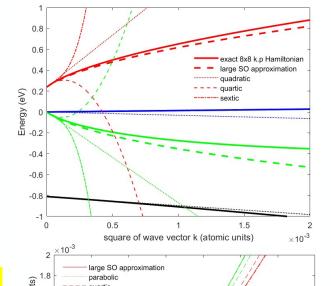
Cubic characteristic equation:

$$\tilde{E}(\tilde{E} - E_0)(\tilde{E} + \Delta_0) - \frac{\hbar^2 k^2 E_P}{2m_0} \left(\tilde{E} + \frac{2\Delta_0}{3}\right) = 0$$

Large spin-orbit approximation:

$$E_{3,4} = \frac{\hbar^2 k^2}{2m_0} + \frac{E_0}{2} \left(1 \pm \sqrt{1 + \frac{\hbar^2 k^2}{2m_0} \frac{2}{\mu_{lh} E_0}} \right)$$

Kane, J. Phys. Chem. Solids 1, 249 (1957).

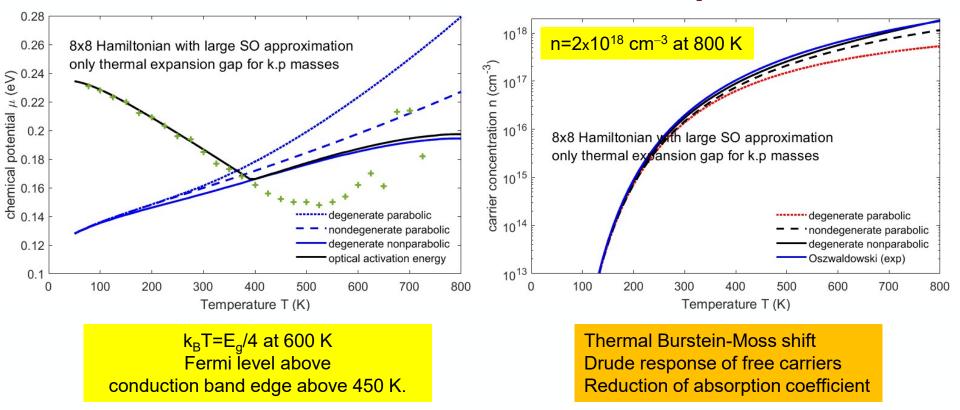


Energy versus k

Density of CB states

$$\frac{\hbar^2 k^2}{2m_0 m^*} = \varepsilon (1 + \alpha \varepsilon + \beta \epsilon^2)$$
$$\alpha = \frac{(1 - m^*)^2}{E_0}$$

Thermal excitations of electron-hole pairs in InSb





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M. Rivero Arias *et al.*, JVSTB **41**, 022203 (2023).

Oswaldowski/Zimpel, J. Phys. Chem. Solids **49**, 1179 (1988). 26 D. L. Rode, Phys. Rev. B **3**, 3287 (1971).

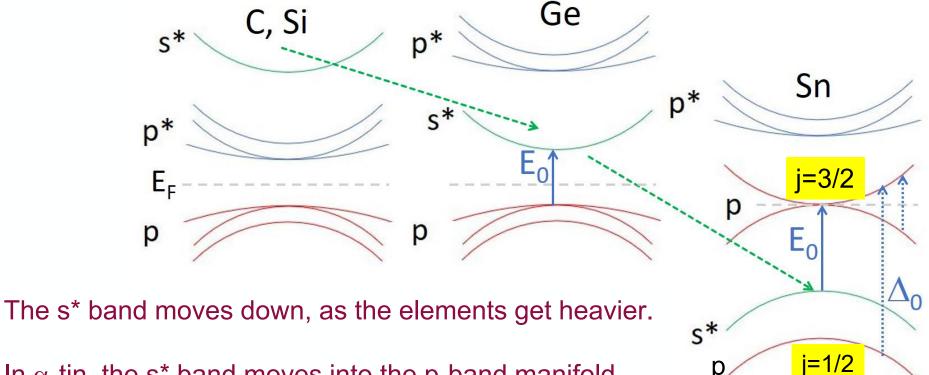
Optical constants model: screened excitons

$$\varepsilon_{2}(E) = \frac{2\pi A\sqrt{R}}{E^{2}} \left\{ \sum_{n=1}^{\sqrt{g}} \frac{2R}{n} \left(\frac{1}{n^{2}} - \frac{n^{2}}{g^{2}} \right) \delta\left[E - E_{0} + \frac{R}{n^{2}} \left(1 - \frac{n^{2}}{g} \right)^{2} \right] + \frac{\sinh(\pi g k) H(E - E_{0})}{\cosh(\pi g k) - \cosh\left(\pi g \sqrt{k^{2} - \frac{4}{g}}\right)} \right\} [f_{h}(E) - f_{e}(E)]$$

- Absorption by screened excitons (Hulthen potential)
- Degenerate Fermi-Dirac statistics to calculate f_h and f_e .
- Numerical Kramers-Kronig transform (need occupation factors)
- Two terms for light and heavy excitons
- Non-parabolicity and temperature-dependent mass included from k.p theory
- k-dependent matrix element P.
- Screening parameter $g=12/\pi^2 a_R k_{TF}$ (large: no screening) Sommerfeld enhancement persists well above the Mott density.
- Only two free parameters: Band gap E_0 and broadening Γ
- Amplitude *A* and exciton binding energy *R* from k.p theory and effective masses



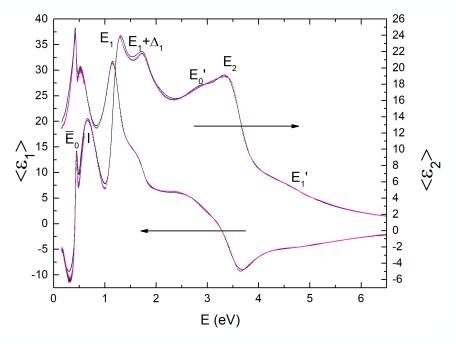
Relativistic Effects: Darwin Shift: C, Si, Ge, Sn

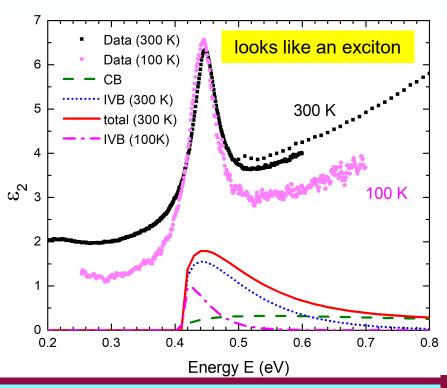


In α -tin, the s* band moves into the p-band manifold, between the j=1/2 and j=3/2 states.

This makes α -tin an (**inverted**) **gapless** semiconductor.

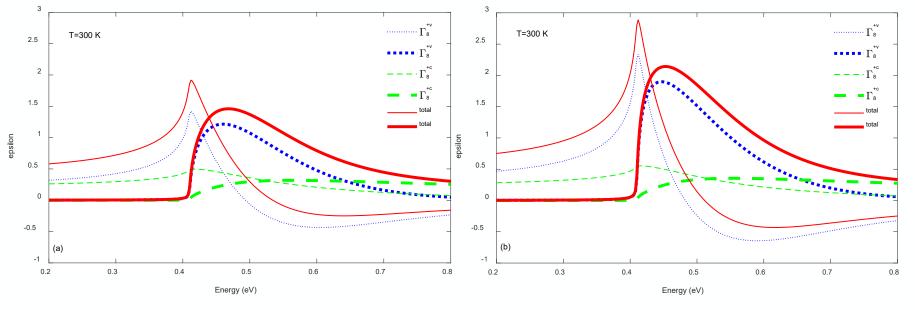
Intravalence band absorption in gapless topological insulators (α -tin)





All gapless (inverted) semiconductors should have this peak. Theory with same model as Ge IVB (Kaiser 1953, Kahn 1955).

Excitonic intravalence band absorption in gapless topological insulators (α -tin)

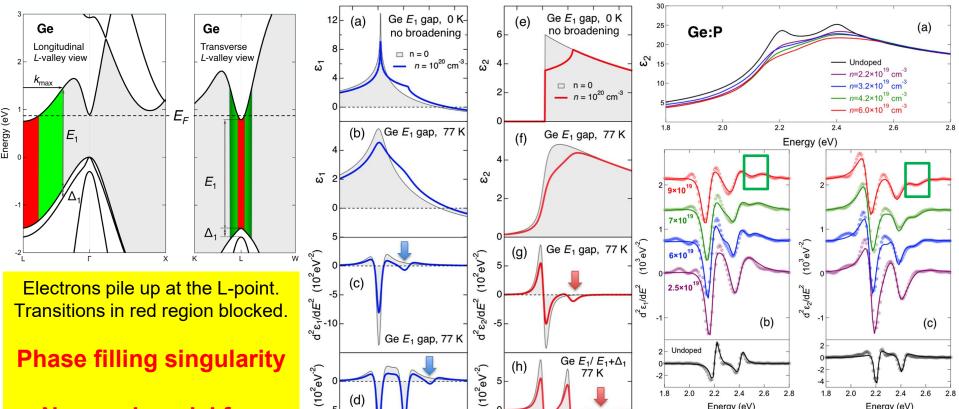


Interband transitions without excitonic effects.

Screened excitons included, parabolic bands. Agreement with experiment is not good.



(2) Highly doped Ge (n-type, with phosphorus)



 $d^2\epsilon_2/dE^2$

2.0

2.2 2.4 2.6 2.8 Energy (eV)

 $Ge^{\parallel}E_1/E_1+\Delta_1$

2.0 2.2 2.4 2.6

Energy (eV)

 $d^2 \varepsilon_1/dE^2$

No good model for 2D exciton screening

Xu et al., PRL 118, 267402 (2017) Xu et al., JAP **125**, 085704 (2019)

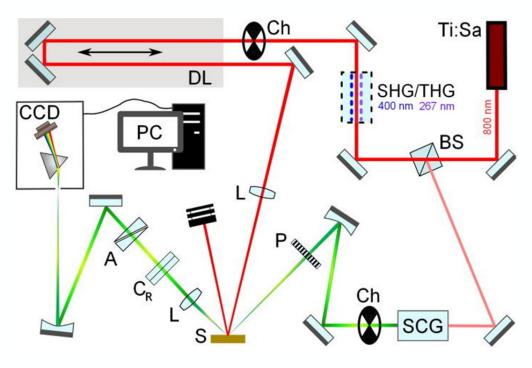
Phase shift

Energy (eV)

31

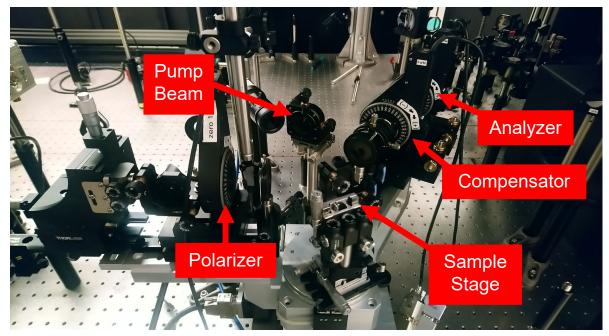
Energy (eV)

(3) Set-up: Femtosecond pump-probe ellipsometry



- Ch: Chopper (500 Hz, 250 Hz)
- A: Analyzer
- P: Polarizer
- C_R: Rotating compensator
- L: Lens
- S: Sample
- DL: Delay Line (~6.67 ns pump-probe delay and 3 fs resolution)
- BS: Beam splitter
- SHG/THG: 2nd/3rd harmonic generation
- SCG: Super-continuum generation
- CCD: Charge-coupled device detector

Set-up: Femtosecond pump-probe ellipsometry



Rotating compensator ellipsometer:

Compensator was rotated in steps of 10° for a total of 55-65 angles.

Probe beam of 350-750 nm at 60° incidence angle.

P-polarized pump beam: 35 fs pulses of 800 nm wavelength at 1 kHz repetition rate.

Delay time from -10 to 50 ps.

Time resolution of about 500 fs.

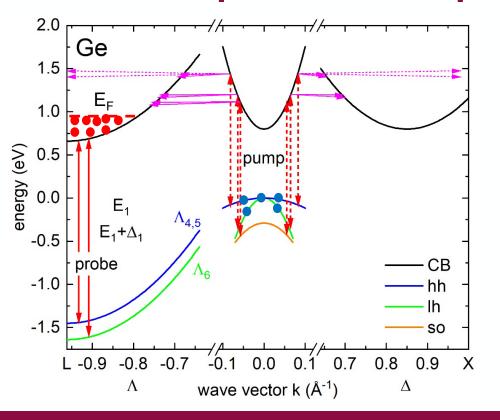


ELI Beamlines: ELI ERIC. Dolní Břežany (near Prague)

Third user call was due October 16th, 2023: https://up.eli-laser.eu/Contact Shirly Espinoza: shirly.espinoza@eli-beams.eu



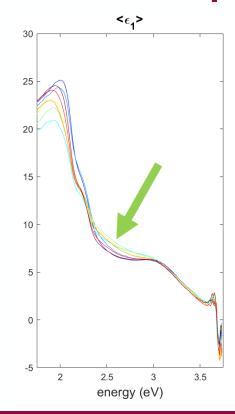
Ultrafast processes in photoexcited Germanium

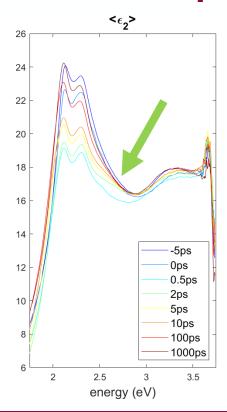


- 1.55 eV pump beam creates N=10²¹ cm⁻³ electron-hole pairs near Γ-point. 9 mW power.
- · Heavy, light, split-off bands.
- Ignore diffusion (for 1.55 eV pump).
- Ignore Auger recombination.
- Thermalization: Fermi-Dirac distribution
- Intervalley scattering: $\Gamma \rightarrow X \rightarrow L$
- Electrons accumulate at L.
- Holes remain near Γ.
- Electrons at L block E₁ and E₁+Δ₁ transitions (Fermi-level singularity)
- Bandgap renormalization: redshift expected
- Lattice heating (25 K): redshift
- Exciton screening



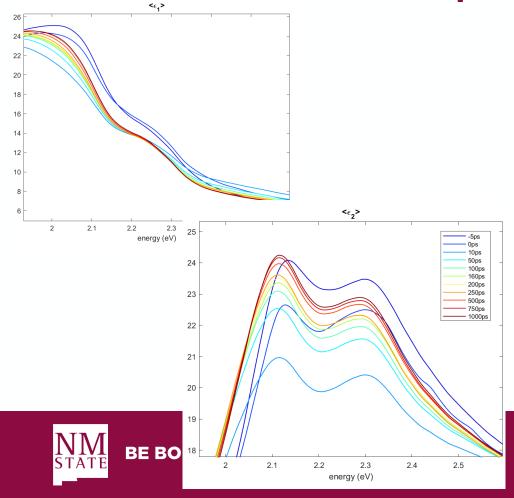
Ultrafast processes in photoexcited Germanium





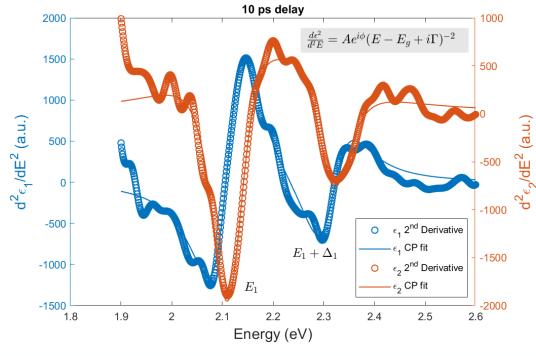
- 1.55 eV pump beam creates N= $4x10^{21}$ cm⁻³ electron-hole pairs near Γ -point.
- The E₁ and E₁+∆₁ peaks decrease within the first two picoseconds and then recover:
 Band filling, excitonic screening followed by diffusion, Auger, and radiative recombination.
- Maybe a Fermi level singularity near 2.6 eV.
- Detailed modeling is required (Tanguy 2-D excitonic line shapes, including Fermi level singularity.)

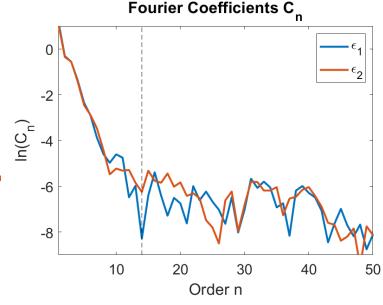
Ultrafast processes in photoexcited Germanium



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- The E₁ and E₁+∆₁ peaks decrease within the first two picoseconds and then recover.
 (Band filling, excitonic screening followed by diffusion, Auger, and radiative recombination)
- Maybe a Fermi level singularity near 2.6 eV.
- It looks like there is a redshift, but we need a line shape analysis with derivatives.

Derivative analysis: critical point parameters

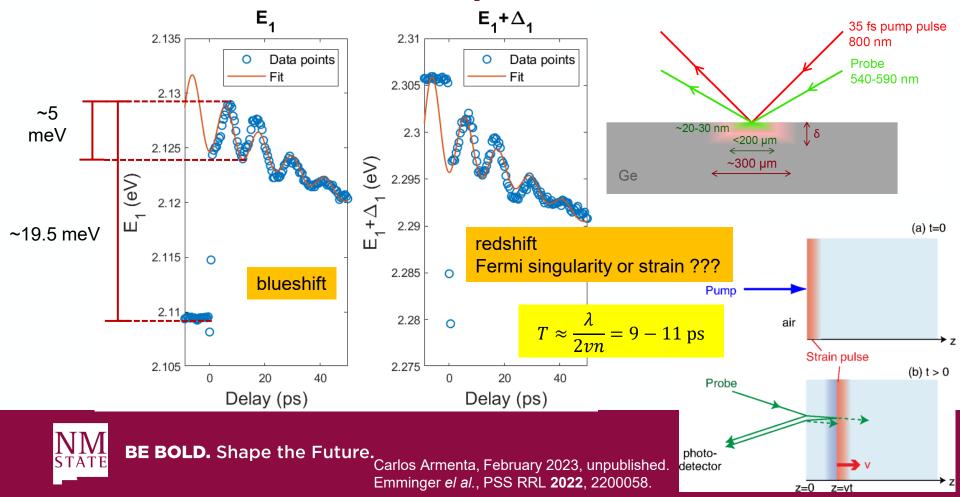




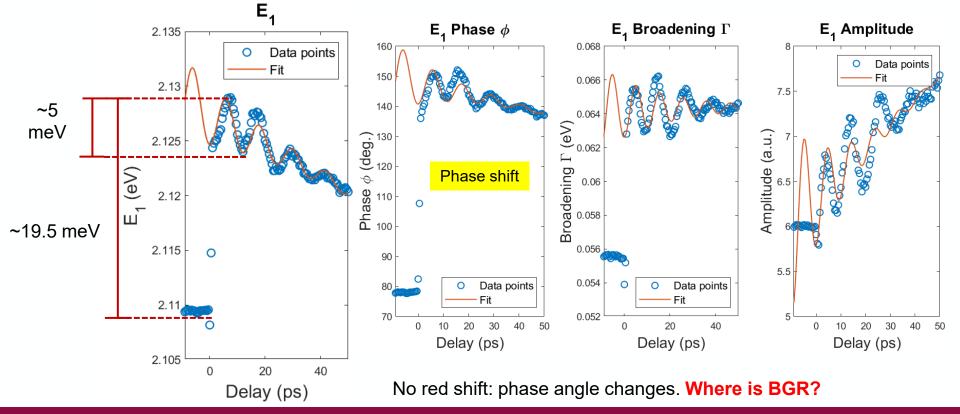
Extended Gauss filter
Filter width determined from Fourier coefficients



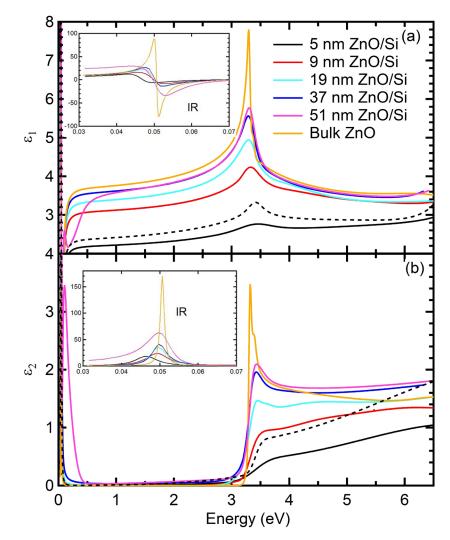
Coherent acoustic phonon oscillations



Critical point parameters for E_1 and $E_1+\Delta_1$

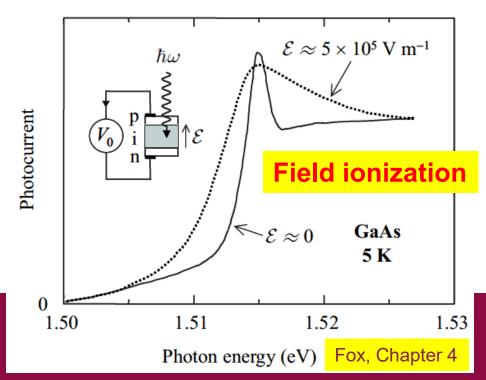






Thickness dependence of excitonic absorption

ZnO on Si with different thicknesses. This might be an electric field effect. (Samarasingha, Sudeshna Chattopadhyay, SZ, JVSTB 2020)



Conclusions

- Quantitative modeling of low-density optical processes is possible with basic physics and matrix elements from k.p theory:
 - Photoluminescence in Ge
 - Indirect gap absorption in Ge
 - Direct gap absorption in Ge at low T
 - More work is needed at high temperatures and for materials other than Ge.
- High carrier excitations:
 - High electron doping density in Ge
 - Thermal excitation of electron-hole pairs in InSb and α -tin.
 - Femtosecond laser generation of electron-hole pairs in Ge (ELI Beamlines)
 - Experimental data and qualitative explanations exist
- We need more experiments and more detailed theory and simulations.





Thank you!

Questions?

Many students contributed to this project.